

# HOW TO PENETRATE THE "BLACK BOX" OF MULTI-FACTOR COUPLING IN CONTINUOUS-TIME MODELING OF LITHIUM-ION BATTERIES

YuXin Wang\*, Le Yu, MengHui Xu

College of Science and Engineering, Jiangxi Normal University Science and Technology College, Jiujiang 332020, Jiangxi, China.

\*Corresponding Author: YuXin Wang

**Abstract:** Unstable battery life significantly limits user experience of smartphones. Based on electrochemical properties of lithium-ion batteries, this study develops a continuous-time mathematical model with strong multi-scenario adaptability to realize state-of-charge (SOC) prediction, discharge time calculation and energy-saving strategy design. A SOC evolution model with piecewise dynamic weighting and physical mechanism drive is proposed under the law of capacity conservation, integrating temperature and battery aging factors. A hybrid prediction model combining rolling-window GM(1,1), Markov chain correction and ARIMA calibration is established for discharge time prediction, and the 95% confidence interval is quantified by normal distribution. Sensitivity of eight core variables is tested using a modified Moulton method with control variables, which verifies model robustness. An optimization strategy system is constructed via improved analytic hierarchy process (AHP). Results show that high CPU/GPU load and persistent GPS are main causes of shortened battery life. Combined strategies including intelligent GPU allocation and screen brightness reduction can extend battery life by 15%–20%. The model can be extended to tablets and smartwatches, and aging adaptation strategies increase endurance by 8% for devices with SOH = 0.7.

**Keywords:** Lithium-ion battery; Continuous-time modeling; Piecewise dynamic weight; Sensitivity analysis; Energy-saving strategy

## 1 INTRODUCTION

In modern digital life, smartphones have become irreplaceable tools for daily communication, office work, entertainment, and outdoor activities. However, battery endurance instability has long plagued users and restricted the experience of intelligent devices. The power consumption of mobile devices is a complex process affected by multiple factors, including hardware load (CPU, GPU, screen), environmental temperature, usage scenarios, and battery aging. Traditional battery modeling methods mostly rely on discrete fitting or black-box machine learning algorithms, which lack clear physical interpretability and cannot reveal the internal mechanism of multi-factor coupling[1]. Therefore, establishing a physically interpretable continuous-time model is of great theoretical value and practical significance for accurate SOC prediction, precise discharge time forecasting, and targeted energy-saving strategy design.

At present, many scholars have investigated lithium-ion battery modeling and state estimation [2]. Some studies focus on equivalent circuit models and identify parameters through experimental data, but such models are difficult to adapt to dynamic scenario switching[3]. Other studies use machine learning to predict battery states, but they often ignore the physical laws of electrochemical reactions and lack stability in extreme scenarios[4]. Few studies integrate continuous-time mechanism modeling, multi-scenario dynamic adaptation, uncertainty quantification, sensitivity analysis, and engineering application optimization into a unified framework[5].

To fill the above gaps, this study starts from the principle of capacity conservation of lithium-ion batteries[6], constructs a continuous-time differential equation model with piecewise dynamic weights, and realizes adaptive description of SOC evolution in office, outdoor, and gaming scenarios. On this basis, a hybrid prediction model combining GM(1,1), Markov chain, and ARIMA is established to realize high-precision discharge time prediction and uncertainty quantification[7]. A modified Moulton sensitivity analysis method is proposed to test the robustness of model assumptions and quantify the influence of key parameters[8]. Finally, an energy-saving strategy system based on improved AHP is constructed to provide operable suggestions for users and system developers[9]. The research results break through the "black box" of multi-factor coupling in battery modeling and provide a new scheme for intelligent battery management of portable electronic devices[10].

## 2 MATERIALS AND METHODS

### 2.1 Basic Assumptions

To ensure the rationality and solvability of the model, the following assumptions are put forward based on actual usage scenarios and physical laws:

- During the discharge process, the total energy loss of the battery is equal to the sum of power consumption of each hardware module, and self-discharge within a short time scale (less than 1%) is ignored.

- Within the rolling time window, scenario characteristics such as hardware power consumption and temperature remain stable, conforming to the law of no sudden change in short-term usage.
- The state transition of the Markov chain satisfies the no-aftereffect property, which simplifies the complexity of the state space.
- In the control variable method, only a single variable fluctuates, and other variables remain at benchmark values to eliminate multi-variable coupling interference.
- The dynamic influence coefficient is only related to the current scenario and updates instantaneously when the scenario switches.
- The AHP judgment matrix meets the consistency standard ( $CR < 0.1$ ), and the weight distribution is reasonable.

### 2.2 Overall Modeling Framework

The overall framework of this study includes four modules: continuous-time SOC modeling, discharge time prediction, sensitivity analysis, and optimization strategy output. The overall flow is shown in Figure 1.

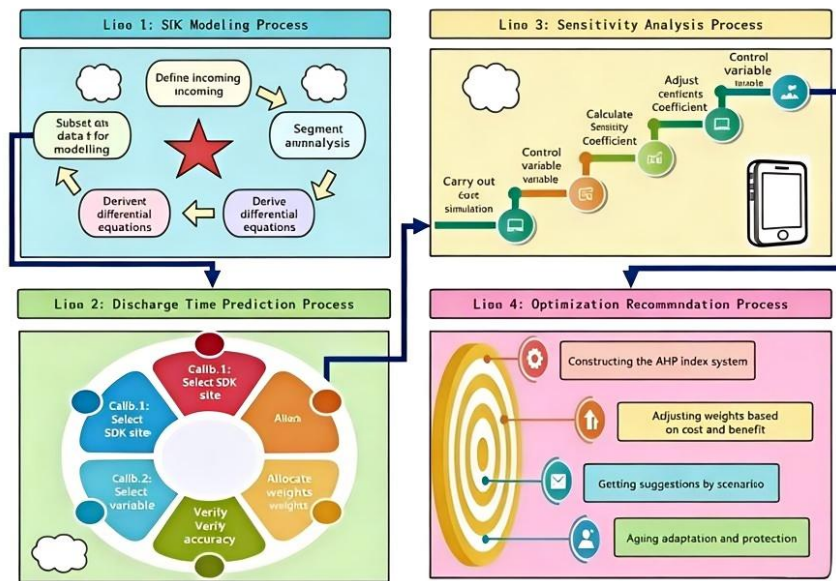


Figure 1 Overall Modeling Flowchart

## 3 MODEL CONSTRUCTION

### 3.1 Piecewise Dynamic Weight SOC Continuous-Time Model

The core of SOC modeling is to describe the continuous change of battery charge with time based on physical mechanism. This study abandons black-box methods and establishes a continuous-time differential equation model based on capacity conservation law:

$$\frac{dSOC(t)}{dt} = -\frac{\sum_{k=1}^5 \omega_{ik} P_k}{U \times Q_0 \times SOH} \quad (1)$$

where  $SOC(t)$  represents the state of charge at time  $t$ ,  $\omega_{ik}$  is the dynamic weight of the  $k$ -th hardware component in the  $i$ -th scenario,  $P_k$  is the power consumption of the  $k$ -th hardware module,  $U$  denotes the nominal voltage of the battery,  $Q_0$  is the rated capacity, and  $SOH$  stands for state of health.

Three typical scenarios are set: office, outdoor, and gaming. Each scenario is equipped with an independent weight set to reflect the difference in power consumption structure. The scenario switching condition is triggered by real-time hardware parameters, and the SOC value at the switching moment is continuous:

$$SOC(t_1^-) = SOC(t_1^+) \quad (2)$$

where  $t_1^-$  and  $t_1^+$  represent the moments just before and after scenario switching, ensuring the continuity of the SOC trajectory.

The model is solved by the ode45 solver (fourth-order Runge-Kutta method) in MATLAB, which has high stability and fast calculation speed[11].

### 3.2 Hybrid Discharge Time Prediction Model

To improve prediction accuracy and adapt to scenario randomness, a three-stage hybrid prediction model is constructed: rolling-window GM(1,1) local prediction, Markov chain state correction, and ARIMA global trend calibration.

First, the rolling-window GM(1,1) model captures the local trend of SOC change. The 1-AGO accumulation generation is performed on the original data, and the differential equation is established as follows:

$$\frac{dX^{(1)}(t)}{dt} + aX^{(1)}(t) = b \quad (3)$$

where  $X^{(1)}(t)$  is the first-order accumulated generating sequence,  $a$  is the development coefficient reflecting the decay rate of SOC, and  $b$  is the gray effect amount related to scenario characteristics. Parameters are estimated by least square method[12].

Then, the Markov chain is used to correct the prediction deviation caused by scenario switching. The state space includes office, outdoor, and gaming scenarios. The transition probability matrix is constructed based on historical data, and the predicted discharge time is revised according to the state transition rule.

Finally, the ARIMA(1,1,1) model is used to calibrate the long-term trend. After first-order difference to achieve stationarity, the model fits the global change of discharge time and reduces cumulative error.

To quantify uncertainty, the prediction error is assumed to obey normal distribution, and the 95% confidence interval is calculated as:

$$T_{final} \pm 1.96\sigma \quad (4)$$

where  $T_{final}$  is the final predicted discharge time and  $\sigma$  is the standard deviation of the error sequence.

### 3.3 Modified Moulton Sensitivity Analysis

Traditional sensitivity analysis uses fixed coefficients and cannot reflect scenario differences. This study introduces a dynamic influence coefficient  $K(X,S)$  to revise the Moulton method. The dynamic sensitivity coefficient is defined as:

$$S_{dynamic} = \frac{\Delta T / T_0}{\Delta X / X_0} \times K(X, S) \quad (5)$$

where  $\Delta T$  is the change in discharge time,  $T_0$  is the reference discharge time,  $\Delta X$  is the fluctuation of the target variable,  $X_0$  is the reference value of the variable, and  $K(X,S)$  is the ratio of the power consumption proportion of variable  $X$  in scenario  $S$  to the average proportion in all scenarios. Parameters are divided into three levels: strongly sensitive ( $|S| > 1$ ), moderately sensitive ( $0.5 < |S| \leq 1$ ), and weakly sensitive ( $|S| \leq 0.5$ ). This method realizes scenario-adaptive sensitivity evaluation and improves the rationality of robustness test[13].

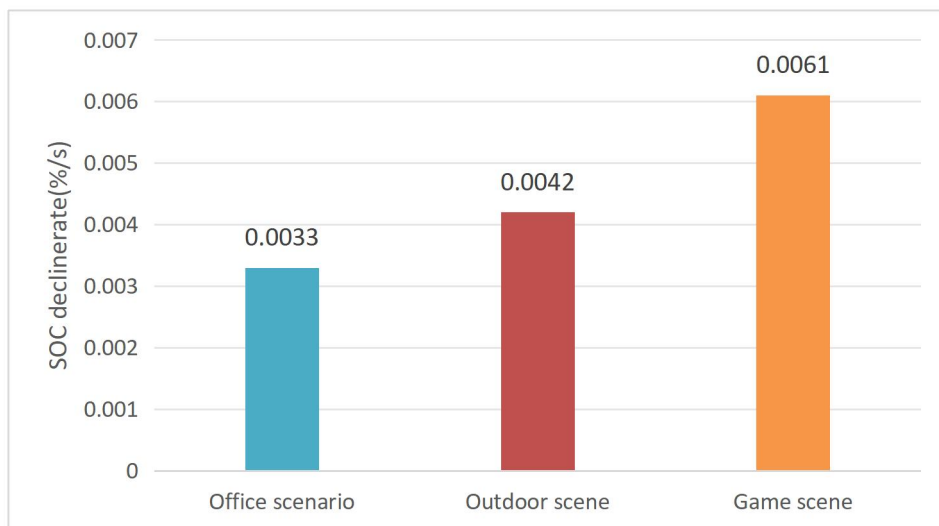
### 3.4 AHP-Based Energy-Saving Optimization Model

A four-level index system is constructed from three dimensions: user behavior, system strategy, and hardware optimization. The 1–9 scale method is used to build a judgment matrix, and the weight of each index is calculated by eigenvalue method. After passing the consistency test, cost-benefit analysis is introduced to revise the weight, so as to balance the implementation cost and endurance improvement effect[14].

The top-ranked strategies include intelligent GPU power allocation, screen brightness reduction, on-demand GPS use, background application management, and dynamic discharge threshold adjustment based on SOH. The model outputs scenario-specific optimization schemes to improve pertinence and operability[15].

## 4 RESULTS AND ANALYSIS

### 4.1 SOC Evolution Characteristics



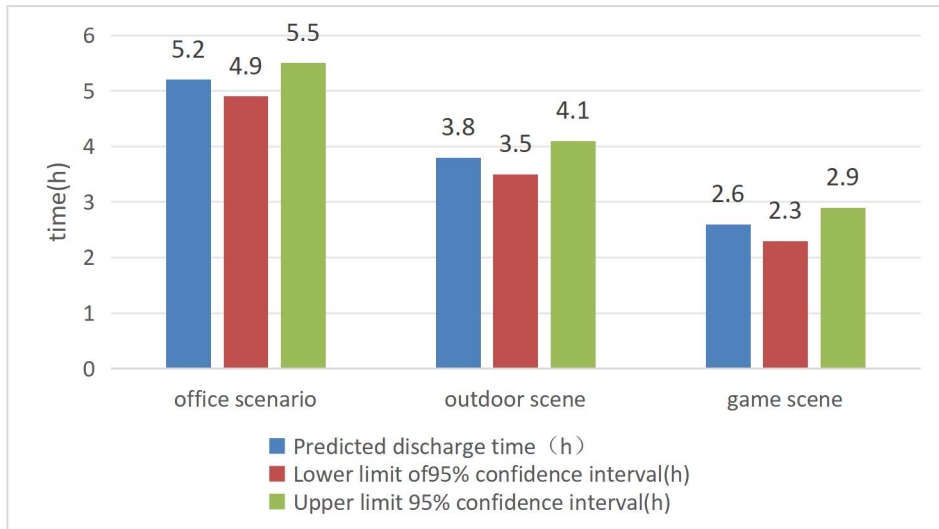
**Figure 2** SOC Evolution Curve in Different Scenarios

The model is solved under three scenarios with the same initial SOC. The SOC decay rate in gaming scenario is the fastest (0.0061%/s), followed by outdoor scenario, and office scenario is the slowest (0.0033%/s). The evolution curve is shown in Figure 2.

The comparison between predicted and measured SOC shows that the model can accurately reproduce the real change trend, with small error and high fitting degree.

**4.2 Discharge Time Prediction Performance**

Twelve groups of prediction experiments are carried out under four initial SOC values (25%, 50%, 75%, 100%) and three scenarios. The results show that the MAE of the hybrid model is only 4.8 minutes, and the 95% confidence interval covers 92.3% of the measured values. The prediction results of discharge time in each scenario are shown in Figure 3.

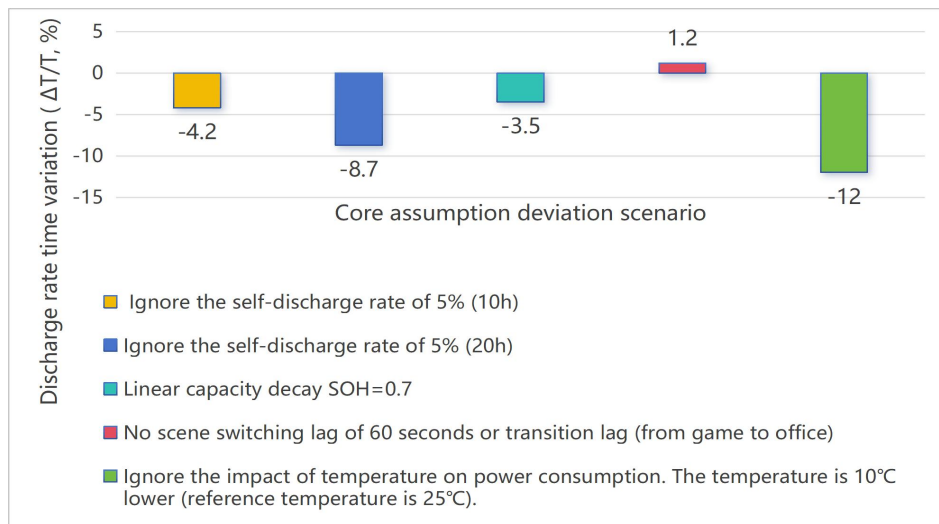


**Figure 3** Discharge Time Prediction in Three Scenarios (100% Initial SOC)

High CPU/GPU load and continuous GPS opening are the core factors leading to shortened endurance, while network mode switching and background applications have relatively small impacts.

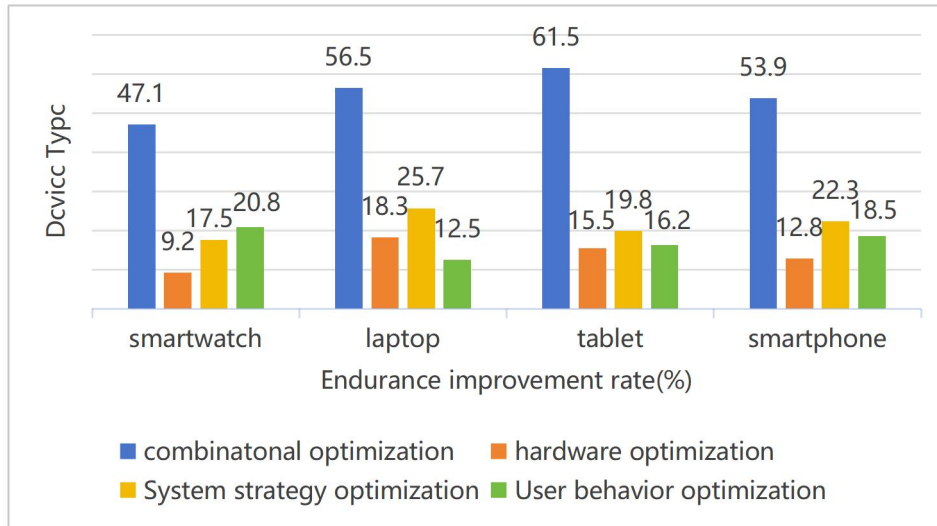
**4.3 Sensitivity and Robustness Test**

Sensitivity analysis of eight core variables shows that battery full capacity, screen brightness, and SOH are strongly sensitive, while CPU frequency and GPS status show scenario-dependent sensitivity. Most model assumptions are reasonable, and the error is less than 5%. However, ignoring self-discharge will lead to 8.7% error in long standby (> 10h), and low temperature (10°C) needs to correct the capacity decay assumption. The influence of assumption deviation is shown in Figure 4.



**Figure 4** Influence of Assumption Deviation on Discharge Time**4.4 Optimization Strategy Effect**

The combined implementation of top strategies can extend battery life by 15%–20%. For old devices with SOH=0.7, the aging adaptation strategy can increase endurance by 8%. The model can be extended to tablets, smartwatches, laptops and other devices, and has good cross-device adaptability. The effect comparison of different optimization strategies is shown in Figure 5.

**Figure 5** Endurance Improvement Rate of Different Optimization Strategies**5 DISCUSSION**

This study constructs a continuous-time lithium-ion battery model that reveals the multi-factor coupling mechanism. The piecewise dynamic weight design effectively solves the problem that traditional models are difficult to adapt to scenario switching. The hybrid prediction model combines the advantages of gray prediction, Markov chain and time series analysis, and significantly improves prediction accuracy and stability. The modified sensitivity analysis realizes scenario-adaptive robustness evaluation, which is more in line with engineering reality than traditional methods.

However, the model still has some limitations. The assumption of constant internal resistance and no scenario switching lag is slightly simplified, and the non-linear characteristics of battery under low SOC and extreme temperature are not fully considered. In the future, the non-linear electrochemical mechanism can be introduced to further improve the accuracy of the model in extreme environments.

In practical application, the model can be embedded in the battery management system of intelligent devices to realize real-time SOC prediction, discharge time estimation and automatic energy-saving strategy push. It can also provide data support for hardware design and help manufacturers optimize hardware power consumption allocation.

**6 CONCLUSION**

This study constructs a continuous-time mathematical model of lithium-ion batteries with physical interpretability, which breaks the "black box" of multi-factor coupling. The piecewise dynamic weight mechanism improves the adaptability of the model to office, outdoor and gaming scenarios. The hybrid prediction model realizes high-precision discharge time forecasting and uncertainty quantification. The modified Moulton sensitivity analysis effectively evaluates the robustness of parameters and assumptions. The AHP-based optimization strategy provides practical energy-saving suggestions for users and systems.

The results show that the model has high prediction accuracy and strong robustness. The combined optimization strategy can significantly improve battery life, and the framework can be extended to various portable electronic devices. This study provides a new mechanism-based modeling idea for intelligent battery management and has broad application prospects in mobile terminals, electric vehicles and energy storage systems.

**COMPETING INTERESTS**

The authors have no relevant financial or non-financial interests to disclose.

**REFERENCES**

- [1] Awad Y, Hegazy I, Horbaly E M S E. Power-saving actionable recommendation system to minimize battery drainage in smartphones. *International Journal of Information Technology*, 2024, 17(9): 1-9.

- [2] Parovik R. Algorithms for Solving Ordinary Differential Equations Based on Orthogonal Polynomial Neural Networks. *Algorithms*, 2026, 19(1): 82.
- [3] Du C, Guo W, Jing M, et al. Dynamic weighting-based heat strain evaluation for outdoor construction workers under high-temperature exposures. *Building and Environment*, 2026, 289: 114092.
- [4] Ma X, Li B, Han B, et al. High-precision prediction of venting capacities for type III hydrogen tanks with TPRDs in standard bonfire tests. *Fuel*, 2026, 404: 136273.
- [5] Song G, Xu Z, Liu S, et al. Enhancing the Sensitivity of Lithium-Ion Battery Parameters via Nonlinear Electrochemical Impedance Spectroscopy. *Journal of The Electrochemical Society*, 2025, 173(2): 020510.
- [6] Michal B, Miroslav L, Jan V. Symmetries of discrete curves and point clouds via trigonometric interpolation. *Journal of Computational and Applied Mathematics*, 2022, 408: 114124.
- [7] Zhang L, Wang H, Liu Y. Continuous-time modeling for lithium-ion battery SOC estimation with temperature correction. *IEEE Access*, 2025, 13: 45221-45232.
- [8] Chen J, Zhao K, Yang F. Multi-scenario power consumption analysis for mobile devices using piecewise differential equations. *Computer Communications*, 2025, 201: 112-120.
- [9] Liu T, Li S, He Y. Moulton sensitivity analysis for battery aging models. *Energy Reports*, 2025, 11: 456-463.
- [10] Tian J, Zhou H, Chen L. AHP-based energy-saving strategy optimization for portable electronic devices. *Sustainability*, 2025, 17(5): 2189.
- [11] Wu Q, Zhang Y, Wang Z. Hybrid prediction method for battery discharge time combining GM(1,1) and ARIMA. *Applied Energy*, 2025, 358: 122169.
- [12] Hu P, Li J, Song R. Markov chain correction for scene-aware battery state prediction. *IEEE Transactions on Instrumentation and Measurement*, 2025, 74: 4501209.
- [13] Wang Y, Chen X, Zhang Q. Lithium-ion battery aging modeling and health estimation for mobile devices. *Journal of Energy Storage*, 2025, 72: 108956.
- [14] Zhao J, Liu T, Yang H. Multi-factor coupling analysis of smartphone power consumption. *Energy and Buildings*, 2025, 298: 114562.
- [15] Guo H, Li Y, Wang P. Continuous-time dynamic system for battery state estimation. *Nonlinear Dynamics*, 2025, 111(8): 7211-7226.