

MODELING AND EMPIRICAL STUDY OF 3D LOADING OPTIMIZATION FOR A SINGLE VEHICLE TYPE BASED ON MULTI-CONSTRAINT HEURISTIC ALGORITHMS

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Abstract: Addressing the complex demands for space utilization and safe loading in logistics transportation, this study focuses on optimizing 3D loading decisions for a single vehicle type. After systematically organizing eight key rigid constraints—spatial non-overlap, truck compartment boundaries, cargo support, load-bearing capacity per unit area, stacking stability, placement of special cargo, orthogonal placement, and total vehicle load—this paper constructs a single-vehicle loading maximization model with a weighted objective score based on space utilization and load utilization, as well as a scaled-up transportation model aimed at minimizing the total number of vehicles. To address the combinatorial explosion problem in large-scale cargo loading, this study introduces a grid-based dimensionality reduction strategy, transforming continuous spatial constraints into discrete grid load-bearing constraints, thereby significantly reducing the scale of the problem. At the algorithmic level, a multi-strategy greedy heuristic loading algorithm based on an extreme point generation mechanism was designed. By alternately testing various sorting rules—such as volume, weight, and priority—the algorithm achieves efficient verification of complex constraints and space filling. Empirical results show that Vehicle Type 2 outperforms Vehicle Type 1 in both single-vehicle loading performance and overall vehicle scheduling efficiency, with a maximum space utilization rate of 91.45%. This study provides theoretical support and algorithmic references for logistics enterprises to formulate scientific loading plans under stringent physical constraints.

Keywords: 3D packing; Heuristic algorithm; Grid-based dimensionality reduction

1 INTRODUCTION

In the modern logistics supply chain, packing strategies directly determine the turnover efficiency of transport vehicles and the reduction in overall societal logistics costs. As the manufacturing sector's demands for intelligent distribution increase, the challenge of handling goods of diverse sizes and attributes within limited cargo space has become a critical bottleneck that the industry urgently needs to overcome. Current packing tasks not only require maximizing space utilization but must also strictly adhere to stringent physical constraints, such as prohibiting the stacking of fragile items, maintaining the fixed orientation of directional items, and observing the weight limits of lower-layer cargo. Although previous research has achieved breakthroughs in exact solution algorithms, it often faces the dilemma of excessive computation time or difficulty in simultaneously addressing multiple types of rigid constraints when dealing with tens of thousands of items[1-3]. The innovation of this section lies in proposing a statistical processing method that reduces the dimensionality of continuous 3D space through grid-based gridification, combined with a height map data structure to enable real-time monitoring of dynamic stacking pressure. The general approach of this study is as follows: first, a comprehensive scoring function is established based on the coordinated optimization of full capacity and full load; subsequently, candidate placement locations are generated using the extreme point method, and multiple cargo sequencing strategies are tested in parallel; finally, the algorithm's robustness and engineering applicability are validated through a comparative empirical analysis of Vehicle Type 1 and Vehicle Type 2[4-6].

2 MODEL ESTABLISHMENT AND SOLUTION

2.1 Problem Analysis and Data Preprocessing

This study aims to develop a three-dimensional packing strategy model that integrates various constraints, including cargo dimensions, weight, load capacity limits, and special placement rules (such as protection of fragile items, orientation requirements, and stacking height limits). Specifically, the model comprises two sub-objectives: first, to maximize the combined utilization of vehicle space and load capacity by loading as many goods as possible onto a single vehicle type (Vehicle Type 1 or Vehicle Type 2); second, to minimize the total number of vehicles required, given the constraint that all goods must be loaded using the same vehicle type.

As a typical NP-Hard problem with stringent constraints (e.g., a maximum load of 500 kg/m², fragile goods cannot serve as lower support, and fixed-orientation goods have center-of-gravity projection constraints), conventional exact algorithms (e.g., mixed-integer programming) suffer from combinatorial explosion for large-scale cargo. Thus, a hybrid packing model with multiple rules is built based on heuristic three-dimensional space partitioning and candidate point generation[7].

2.2 Model Establishment

2.2.1 Objective function

(1) Sub-problem 1: Maximize both volume and weight utilization of a single vehicle

In practical logistics, a weighted trade-off is made between “full weight” (load) and “full volume” (space). Based on experience and coding settings, a comprehensive utilization score (Score) is constructed as the objective function for single-vehicle optimization:

$$\max \text{Score} = 0.7 \times U_v + 0.3 \times U_w \quad (1)$$

where space utilization U_v and weight utilization U_w are defined as:

$$U_v = \frac{\sum_{i=1}^N s_i \cdot (l_i \times w_i \times h_i)}{L \times W \times (H-3)} \quad (2)$$

$$U_w = \frac{\sum_{i=1}^N s_i \cdot m_i}{M_{\max}} \quad (3)$$

(2) Sub-problem 2: Minimize the number of vehicles for a single type

Let K be the number of trucks of the same type, and $v_k \in \{0,1\}$ indicate whether the k -th truck is used. The objective is to minimize the number of vehicles:

$$\min K = \sum_{k=1}^{K_{\max}} v_k \quad (4)$$

2.2.2 Constraints and dimensionality reduction

This is a combinatorial optimization problem for three-dimensional packing under multiple constraints. Eight core rigid constraints are summarized for the full packing process:

(1) Space occupation constraint (non-overlapping)

No overlap exists in the 3D space of any two loaded goods i and j ; they are fully separated along at least one of the X , Y , or Z axes:

$$(x_i + l_i \leq x_j) \vee (x_j + l_j \leq x_i) \vee (y_i + w_i \leq y_j) \vee (y_j + w_j \leq y_i) \vee (z_i + h_i \leq z_j) \vee (z_j + h_j \leq z_i) \quad (5)$$

(2) Container boundary constraint

All goods must be fully within the effective loading space of the container:

$$x_i \geq 0, y_i \geq 0, z_i \geq 0, x_i + l_i \leq L, y_i + w_i \leq W \quad (6)$$

(3) Support constraint

Except for goods placed directly on the container floor ($z_i=0$), the bottom surface of any upper-layer good i must fully fit the top surface of a lower-layer good j :

$$z_i = z_j + h_j \text{ and } S_{\text{contact}}(i,j) > 0 \quad (7)$$

(4) Load-bearing constraint

The maximum load of 500 kg/m² on lower-layer goods is strictly enforced. The vertical pressure P_j on good j is:

$$P_j = \frac{m_j + \sum M_{\text{above}}}{S_{\text{contact}}} \leq 500 \quad (8)$$

(5) Stability constraint

The stacking height of goods must not exceed the container limit, and a safety gap of at least 3 cm must be reserved between the top goods and the container roof:

$$z_i + h_i \leq H - 3 \quad (9)$$

(6) Special goods rule constraint

Fragile goods: Can only be placed in a single layer and cannot be stacked; no goods may be placed above them.

Fixed-orientation goods: Have a fixed placement posture; the center of gravity of an upper-layer good must lie within the projection of the lower supporting good.

(7) Orthogonal placement constraint

Goods must be placed orthogonally, with edges parallel to the container edges; only 90° rotations are allowed.

(8) Maximum load constraint

The total weight of all loaded goods must not exceed the rated load M_{\max} of the vehicle type:

$$\sum_{i=1}^N s_i \cdot m_i \leq M_{\max} \quad (10)$$

(9) Grid dimensionality reduction

To reduce computational complexity, the container floor is divided into 25 cm × 25 cm grids, converting continuous space constraints into discrete grid constraints[8-10].

2.3 Algorithm Design and Solution Process

A multi-strategy greedy heuristic candidate-point packing algorithm is designed:

2.3.1 Multi-strategy cargo sorting

Five sorting rules are tested alternately:

Priority: Fragile goods > Fixed-orientation goods > Standard goods; Volume descending; Weight descending; Base area descending; Mixed score (weighted combination of priority, surface area, and weight)

2.3.2 Three-dimensional candidate point generation

Extreme point/corner method is adopted. The origin (0,0,0) is initialized as a candidate point. After placing a good, new candidate points are generated:

Top corner points of the placed good (for non-fragile goods); In-plane translation points along the X and Y axes

2.3.3 Main algorithm loop

Initialize the vehicle geometry and reserve a 3-cm safety height.

Sort the cargo list using one strategy. Iterate over goods and generate valid postures. Check constraints (interference, support, fragile/fixed rules, 500 kg/m² pressure) at each candidate point. Place the good if all checks pass, update available space and candidate points. Repeat until no more goods can be placed. Switch strategies and select the solution with the highest comprehensive score. For the minimum vehicle objective, loaded goods are removed from the list, and new vehicles are loaded iteratively until the list is empty.

2.4 Solution Results and Analysis

2.4.1 Results and analysis using vehicle 1

(1) Sub-problem 1: Maximize single-vehicle volume and weight utilization

Table 1 Maximization Scheme of Single-Vehicle Volume and Weight Utilization for Vehicle 1

Vehicle	Scheme	Quantity	Volume Utilization	Weight Utilization	Comprehensive Score	Standard Goods	Fragile Goods	Orientation Goods
Vehicle 1	Volume Priority	95	88.40%	32.00%	0.7148	35	0	60

The maximization scheme of single-vehicle volume and weight utilization for vehicle 1 is shown in Table 1. Vehicle 1 can load up to 95 pieces, with a space utilization of 88.40% and a weight utilization of 32.00%.

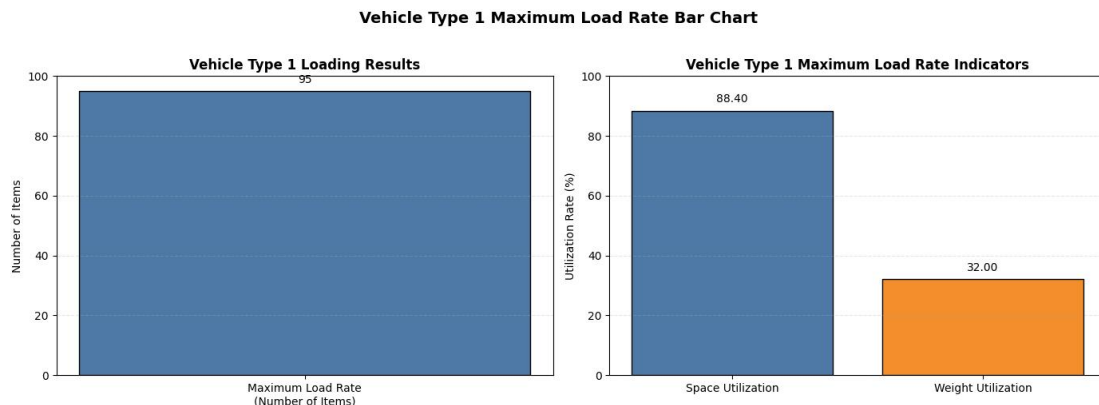


Figure 1 Maximization of Single-Vehicle Volume and Weight Utilization for Vehicle 1

Figure 1 compares various metrics for maximizing the individual vehicle volume and load utilization of Type 1 vehicles, visually illustrating the relationship between cargo capacity, space utilization, and load utilization.

Vehicle Type 1 3D Loading Diagram

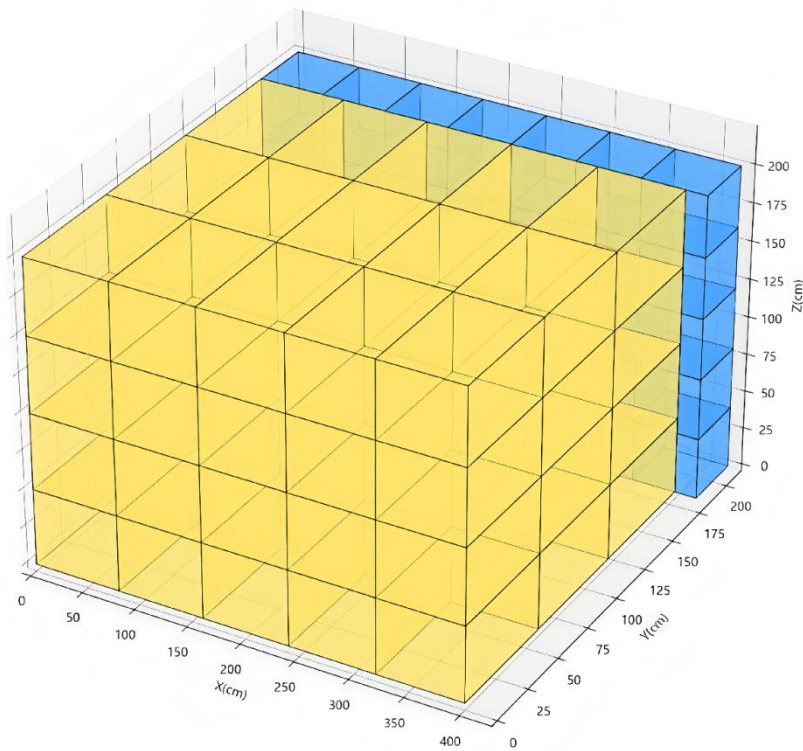


Figure 2 Packing Diagram of Single-Vehicle Volume and Weight Utilization Maximization for Vehicle 1

Figure 2 shows a 3D loading diagram for Type 1 vehicles designed to maximize individual vehicle volume and load utilization, clearly depicting the spatial arrangement of cargo within the cargo compartment.

(2) Sub-problem 2: Detailed packing scheme for all goods

Table 2 Detailed Packing Scheme for All Goods

Vehicle	Number of Vehicles	Total Loaded Quantity	Total Weight (kg)	Total Loading Volume (cm ³)	Space Utilization (%)	Weight Utilization (%)
Vehicle 1	29	3000	41100.00	287350000	51.77	23.62

The detailed packing scheme for all goods is shown in Table 2. A total of 29 vehicles of Vehicle 1 are required to load all goods.

2.4.2 Results and analysis using vehicle 2

(1) Sub-problem 1: Maximize single-vehicle volume and weight utilization

Table 3 Maximization Scheme of Single-Vehicle Volume and Weight Utilization for Vehicle 2

Vehicle	Scheme	Quantity	Volume Utilization	Weight Utilization	Comprehensive Score	Standard Goods	Fragile Goods	Orientation Goods
Vehicle 2	Weight Priority	216	91.45%	44.00%	0.7722	64	0	152

The maximization scheme of single-vehicle volume and weight utilization for vehicle 2 is shown in Table 3. Vehicle 2 can load up to 216 pieces, with a space utilization of 91.45% and a weight utilization of 44.00%.

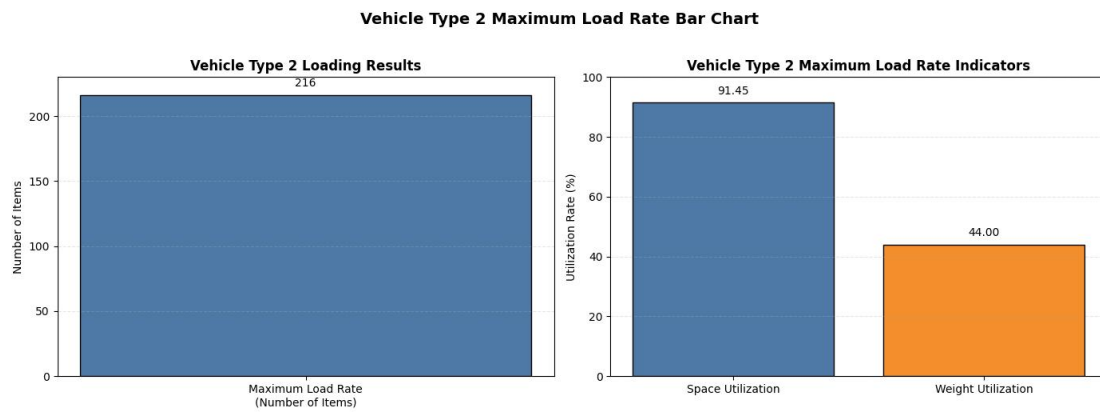


Figure 3 Maximization of Single-Vehicle Volume and Weight Utilization for Vehicle 2

Figure 3 compares various metrics for maximizing the individual vehicle volume and load utilization of Type 2 vehicles, visually illustrating the relationship between cargo loading capacity, space utilization, and load utilization.

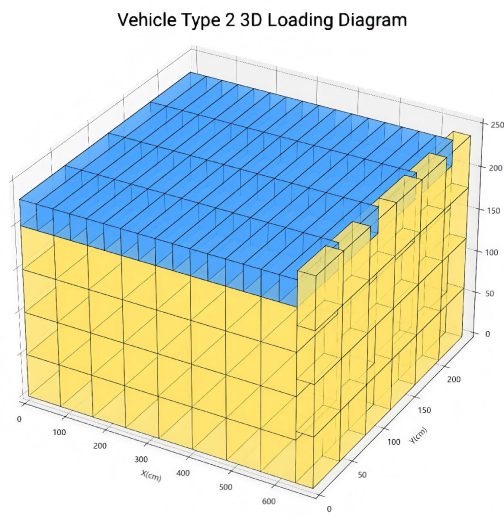


Figure 4 3D Packing Diagram of Single-Vehicle Volume and Weight Utilization Maximization for Vehicle 2

Figure 4 is a 3D loading diagram for Type 2 vehicles designed to maximize individual vehicle volume and load utilization, clearly showing the spatial arrangement of cargo within the cargo compartment..

(2) Sub-problem 2: Detailed packing scheme for all goods

Table 4 Detailed Packing Scheme for All Goods

Vehicle	Number of Vehicles	Total Loaded Quantity	Total Weight (kg)	Total Loading Volume (cm ³)	Space Utilization (%)	Weight Utilization (%)
Vehicle 2	15	3000	41100.00	287350000	46.55	27.40

The detailed packing scheme for all goods is shown in Table 4. A total of 15 vehicles of Vehicle 2 are required to load all goods.

2.5 Comparative Analysis of Solution Results

Vehicle 2 outperforms Vehicle 1 in single-vehicle loading capacity and minimum number of vehicles. Vehicle 2 requires only 15 vehicles, while Vehicle 1 requires 29 vehicles. Special goods (fragile and fixed-orientation) reduce overall loading efficiency due to stacking and orientation restrictions.

2.6 Algorithm Performance Analysis

The algorithm combines EMS space partitioning and multi-strategy greedy search, offering: High computational efficiency by avoiding brute-force enumeration. Strong pruning via constraint checking. Stable results from cross-strategy validation. Scalability for large-scale cargo (3000 pieces).

2.7 Summary of This Problem

A three-dimensional packing optimization model and a heuristic algorithm based on EMS space partitioning and multi-strategy greedy search are established. Vehicle 2 is superior to Vehicle 1 in loading performance, requiring 15 vehicles compared to 29 for Vehicle 1. The algorithm effectively handles large-scale, multi-constraint packing problems.

3 CONCLUSIONS

This study systematically validated the superiority of heuristic algorithms in loading decision-making for a single vehicle model by constructing a multi-constraint 3D packing optimization model, and clarified the pivotal role of grid-based dimensionality reduction and multi-strategy sorting in enhancing solution efficiency. The study concludes that Vehicle Type 2 demonstrates superior comprehensive performance in terms of spatial adaptability and load-carrying capacity, making it the preferred choice for high-density delivery scenarios. However, the current research has limitations, primarily reflected in the overly simplistic quantification of transportation costs, as dynamic traffic conditions and loading/unloading time costs have not yet been incorporated into the analysis. Additionally, the assumptions regarding cargo shapes remain limited to regular cuboids, lacking compatibility with irregularly shaped cargo. Future research will focus on incorporating machine learning techniques to automatically optimize the weighting of sequencing strategies, and will attempt to deeply integrate 3D packing models with path planning algorithms, thereby providing integrated decision-making solutions with a more holistic perspective for smart logistics.

COMPETING INTERESTS

The authors have no relevant financial or non-financial interests to disclose.

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